CLASS WORK - UNDERSTANDING

1. If $f'(a^+) = \lim_{x\to a^+} \frac{f(x)-f(a)}{x-a}$ & $f'(a^-) = \lim_{x\to a^-} \frac{f(x)-f(a)}{x-a}$ then find $f'(a^+)$ & $f'(a^-)$ for the following functions and comment about existence of $f'(a)$.

(i) $f(x) = 2x^2 - x + 3$, where a = 2
(ii) $f(x) = |\ln(x)|$, where a = 1
(iii) $f(x) = e^{-|x|}$, where a = 0
(iv) $f(x) = \cos x$, where a = 0
(v) $f(x) = \begin{cases} x^2+1, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$, where a = 0
(vi) $f(x) = \begin{cases} (x-e)\cdot 2^{-2/(e-x)} & x \neq e \\ 0, & x = e \end{cases}$, where a = e

2. The slope of the curve y = f(x) at the point $P(x_o, f(x_o))$ is the number $m = \lim_{h\to 0} \frac{f(x_o+h)-f(x_o)}{h} = Exist$.

The tangent line to the curve at P is the line through P with given slope m.

Now, answer the following question based on above theory.

(i) Find the slope of the curve y = 1/x at any point x = a ≠ 0. What is the slope at the point x = - 1?
(ii) Where does the slope equals – 1/4?
(iii) Write equation of tangent at P(2, 1/2).

3. Consider $\nabla y = \frac{\Delta y}{\Delta x}$ *or gradient* $= \frac{y_2 - y_1}{x_2 - x_1}$, where y = f(x). Select the correct options.

(A) $\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$.

(B) $f'(x_1) = \lim_{x\to x_1} \frac{y-y_1}{x-x_1}$ and point $P(x_1, y_1)$ lies on the curve y = f(x).

(C) Greater gradient implies greater rate of change.

(D) Equation of tangent at point $P(x_1, y_1)$ lies on the curve y = f(x) is: $y - y_1 = \left.\frac{dy}{dx}\right|_{at\,P} (x - x_1)$.

4. Explain the geometrical meaning of f′(a) and discuss its different aspect of differentiability of f(x) at x = a.

5. Differentiate the following functions using first principal.

(i) $f(x) = \frac{x}{x-1}$
(ii) $f(x) = \sqrt{x}$ for x > 0.
(iii) $f(x) = x^n$, n ∈ Q.
(iv) f(x) = ln(1 + x)
(v) tan(√x)
(vi) $f(x) = \tan^{-1}(x)$
(vii) $f(x) = e^{\sqrt{3x+2}}$
(viii) f(x) = ln(3x + 2), also find f′(0).

6. When does f(x) said to be non-differentiable at x = a (finite). Select the correct options and Justify using an appropriate Example?

(A) If $f'(a^+) \neq f'(a^-)$ (both finite).
(B) If $f'(a^+) = \infty$ & $f(a^-) = -\infty$ or $f'(a^+) = -\infty$ & $f(a^-) = \infty$.
(C) If f(x) is discontinuous at x = a.
(D) f(x) has vertical tangent at x=a. Ex $f(x)=x^{1/3}$ at x=0

7. (i) Let f(x) be a function satisfying $|f(x)| \leq x^2$ for $-1 \leq x \leq 1$. Show that f is differentiable at x = 0 and find f′(0).

(ii) Show that $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ is differentiable at x = 0 and find f′(0).

8. Prove that differentiability of f(x) at x = a implies continuity at x = a but converse is not true.

9. Explain the differentiability of f(x) in x ∈ (a, b) or x ∈ [a, b] and check the differentiability of $f(x)=\begin{cases} |1-4x^2|, & 0\leq x<1 \\ [x^2-2x], & 1\leq x\leq 2 \end{cases}$ in x ∈ (0, 2). Where [.] = GIF & |.| = Modulus function.

10. If $f(x)=\begin{cases} ax+b, & x\leq -1 \\ ax^3+x+2b & x>-1 \end{cases}$ is differentiable for all x ∈ R. Find 'a' & 'b'.

11. If $f(x)=\begin{cases} x^m\cdot\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$ is continuous but not differentiable at x = 0, then find m.

12. If $f(x)=\begin{cases} \sqrt{4x^2-12x+9}\cdot\{x\}, & x\geq 1 \\ \cos\left(\frac{\pi(|x|-\{x\})}{2}\right), & x<1 \end{cases}$ then check the differentiability in [-1, 2].

13. Prove that $f'(x) = u(x)\cdot v'(x) + u'(x)\cdot v(x)$ where $f(x) = u(x)\cdot v(x)$.

14. Let $f(x) = 15 - |x - 10|$; x ∈ R. Then the set of all values of x, at which the function, g(x) = f(f(x)) is not differentiable, is:

(A) {5, 10, 15} (B) {10, 15} (C) {5, 10, 15, 20} (D) {10}

15. Let S = {t ∈ R: $f(x) = |x - \pi|(e^{|x|}-1)\sin|x|$ is not differentiable at t}. Then the set S is equal to

(A) {0} (B) {π} (C) {0, π} (D) ∅ (an Empty set)

16. For x ∈ R, f(x) = |log2 - sinx| and g(x) = f(f(x)), then:

(A) g′(0) = - cos(log2)
(B) g is differentiable at x = 0 and g′(0) = - sin(log2).
(C) g is not differentiable at x = 0
(D) g′(0) = cos(log2)

17. If the function $g(x)=\begin{cases} k\sqrt{x+1}, & 0\leq x\leq 3 \\ mx+2, & 3<x\leq 5 \end{cases}$ is differentiable, then the value of k + m is

(A) 10/3 (B) 4 (C) 2 (D) 16/5

18. Let f:R→R and g:R→R be respectively given by f(x) = |x| + 1 and g(x) = x^2 + 1. Define h:R →R by $h(x)=\begin{cases}\max\{f(x),g(x)\} & if\ x\le 0\\ \min\{f(x),g(x)\} & if\ x>0\end{cases}$. The number of points at which h(x) is not differentiable is ____

19. If |c| ≤ $\frac{1}{2}$ and f(x) is a differentiable function at x = 0 given by $f(x)=\begin{cases} b\sin^{-1}\left(\frac{c+x}{2}\right), & -\frac{1}{2}<x<0\\ 1/2, & x=0\\ \frac{e^{ax/2}-1}{x}, & 0<x<\frac{1}{2}\end{cases}$.

Find the value of 'a' and prove that 64 $b^2 = 4 - c^2$.

20. The left hand derivative of f(x) = [x]sin(πx) at x = k, k an integer, is
(A) $(-1)^k (k - 1)\pi$ (B) $(-1)^{k-1}(k - 1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1} k\pi$

FUNCTIONAL RELATIONSHIP

1. Let f be a differentiable function satisfying $f\left(\frac{x}{y}\right)=f(x)-f(y)$ for all x, y > 0. If f′(1) = 1 then find f(x).

2. A differentiable function satisfying the relation $f(x+y)=f(x)+f(y)+2xy-1\,\forall x,y\in R$. If $f'(0)=\sqrt{3+a-a^2}$ find f(x) and prove that f(x) > 0 ∀ x ∈ R.

3. If f(x + y) = f(x)·f(y), ∀ x, y ∈ R then prove that $f(kx) = f(x)^k$ for ∀ k, x ∈ R.

4. Let f:R→(- π, π) be differentiable function such that f(x) + f(y) = $f\left(\frac{x+y}{1-xy}\right)$. If $f(1)=\frac{\pi}{2}$ and $\lim_{x\to 0}\frac{f(x)}{x}=2$, find f(x).

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ANSWER KEY & SOLUTION

1. (i) Exist, f′(2) = 7 (ii) DNE, f′(1^+) = 1, f′(1^-) = - 1
   (iii) DNE, f′(0^+) = -1, f′(0^-) = 1 (iv) Exist, f′(0) = 0
   (v) DNE, f′(0^+) = 0, f′(0^-) = ∞ (vi) DNE, $f'\left(e^{+}\right)=\infty, f'\left(e^{-}\right)=0$
2. (i) – 1/a^2, - 1 (ii) a = ± 2 (iii) x + 4y – 4 = 0
3. A, B, C
5. (i) $f'(x)=-\frac{1}{(x-1)^2}$ (ii) $f'(x)=\frac{1}{2\sqrt{x}}$
   (iii) $f'(x)=n\cdot x^{n-1}$ (iv) $f'(x)=\frac{1}{1+x}$
   (v) $f'(x)=\sec^2\left(\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}$ (vi) $f'(x)=\frac{1}{1+x^2}$
   (vii) $f'(x)=e^{\sqrt{3x+2}}\times\frac{1}{2\sqrt{3x+2}}\times 3$ (viii) $f'(x)=\frac{3}{3x+2}, f'(0)=\frac{3}{2}$
6. A, B, C, D
7. (i) 0 (ii) 0
9. At x = 1/2, 1
10. a = -1/2, b = 1
11. m ∈ (0, 1]
12. Non-differentiability at x = 0, 1, 3/2, 2
14. A
15. D
16. D
17. C
18. 3
19. a = 1
20. A

FUNCTIONAL RELATIONSHIP

1. f(x) = ln(x)
2. $f(x)=x^2+\left(\sqrt{3+a-a^2}\right)x+1$
3. $f(x)=e^{\lambda x}$
4. $f(x)=2\tan^{-1}x$